

AN S -BRANE SOLUTION WITH ACCELERATION AND SMALL ENOUGH VARIATION OF G

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An S -brane solution with two non-composite electric branes and a set of l scalar fields is considered. The intersection rule for branes corresponds to the Lie algebra A_2 . The solution contains five factor spaces with the fifth one interpreted as “our” 3-dimensional space. It is shown that there exists a time interval where accelerating expansion of “our” 3-dimensional space is compatible with small enough value of effective gravitational “constant” variation.

1. Introduction

As is well known [1, 2, 3, 4, 5, 6, 7], cosmological models in scalar-tensor and multidimensional theories are a framework for describing possible time variations of fundamental physical constants due to scalar fields which are present explicitly in STT or are generated by extra dimensions in multidimensional theories. In [8], we have obtained solutions for a system of conformal scalar and gravitational fields in 4D and calculated the presently possible relative variation of G at the level of less than $10^{-12} yr^{-1}$.

Later, in the framework of a multidimensional model with a perfect fluid and two factor spaces (our 3D space of Friedmann open, closed and flat models and an internal 6D Ricci-flat space) we have obtained the same limit for such variation of G [9].

We have also estimated the possible variations of the gravitational constant G in the framework of a generalized (Bergmann-Wagoner-Nordtvedt) scalar-tensor theory of gravity on the basis of field equations, without using their special solutions. Specific estimates were essentially related to values of other cosmological parameters (the Hubble and acceleration parameters, the dark matter density etc.), but the values of \dot{G}/G compatible with modern observations did not exceeded 10^{-12} per year [10].

In [11], we continued the studies of models in arbitrary dimensions and obtained relations for \dot{G} in a multidimensional model with a Ricci-flat internal space and a multicomponent perfect fluid. A two-component example, dust + 5-brane, was considered explicitly. It was shown that \dot{G}/G is smaller than $10^{-12} yr^{-1}$. Expressions for \dot{G} were also considered in a multidimensional model with an Einstein internal space and a multicomponent perfect fluid [12]. In the case of two factor spaces

with non-zero curvatures without matter, a mechanism for prediction of small \dot{G} was suggested. The result was compared with our exact (1+3+6)-dimensional solutions obtained earlier.

A multidimensional cosmological model describing the dynamics of $n + 1$ Ricci-flat factor spaces M_i in the presence of a one-component anisotropic fluid was considered in [13]. The pressures in all spaces were supposed to be proportional to the density: $p_i = w_i \rho$, $i = 0, \dots, n$. Solutions with accelerated power-law expansion of our 3-space M_0 and a small enough variation of G were found. These solutions exist for two branches of the parameter w_0 . The first branch describes super-stiff matter with $w_0 > 1$, the second one may contain phantom matter with $w_0 < -1$, e.g., when G grows with time.

Similar exact solutions, but nonsingular ones and with an exponential behaviour of the scale factors, were considered in [14] for the same multidimensional cosmological model describing the dynamics of $n + 1$ Ricci-flat factor spaces M_i in the presence of a one-component perfect fluid. Solutions with accelerated exponential expansion of our 3-space M_0 and small variation of G were also found.

Here we continue our investigations of \dot{G} in multidimensional cosmological models. The main problem is to find an interval of the synchronous time τ where the scale factor of our 3D space exhibits an accelerated expansion according to the observational data [15, 16] while the relative variation of the effective 4-dimensional gravitational constant is small enough as compared with the Hubble parameter, see [17, 18, 12, 19, 20] and references therein.

As we have already mentioned, in the model with two non-zero curvatures [12] there exists an interval of τ where accelerated expansion of “our” 3-dimensional space co-exists with a small enough value of \dot{G} . In this paper we suggest an analogous mechanism for a model

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with two form fields and several scalar fields (e.g., phantom ones).

2. The model

We here deal with S -brane solutions describing two electric branes and a set of l scalar fields.

The model is governed by the action

$$S = \int d^D x \sqrt{|g|} \left\{ R[g] - h_{\alpha\beta} g^{MN} \partial_M \varphi^\alpha \partial_N \varphi^\beta - \sum_{a=1,2} \frac{1}{N_a!} \exp[2\lambda_a(\varphi)] (F^a)^2 \right\}. \quad (2.1)$$

Here $g = g_{MN}(x) dx^M \otimes dx^N$ is the D -dimensional metric of pseudo-Euclidean signature $(-, +, \dots, +)$, $F^a = dA^a$ is a form of rank N_a , $(h_{\alpha\beta})$ is a non-degenerate symmetric matrix, $\varphi = (\varphi^\alpha) \in \mathbb{R}^l$ is a vector of l scalar fields, $\lambda_a(\varphi) = \lambda_{a\alpha} \varphi^\alpha$ is a linear function, with $a = 1, 2$ and $\alpha, \beta = 1, \dots, l$, and $|g| = |\det(g_{MN})|$.

We consider the manifold

$$M = (0, +\infty) \times M_1 \times M_2 \times M_3 \times M_4 \times M_5. \quad (2.2)$$

where M_i are oriented Riemannian Ricci-flat spaces of dimensions d_i , $i = 1, \dots, 5$, and $d_1 = 1$.

Let two electric branes be defined by the sets $I_1 = \{1, 2, 3\}$ and $I_2 = \{1, 2, 4\}$. They intersect on $M_1 \times M_2$. The first brane also covers M_3 while the second one covers M_4 . The first brane corresponds to the form F^1 and the second one to the form F^2 .

For the world-volume dimensions of branes we get

$$d(I_s) = N_s - 1 = 1 + d_2 + d_{2+s}, \quad (2.3)$$

$s = 1, 2$, and

$$d(I_1 \cap I_2) = 1 + d_2 \quad (2.4)$$

is the brane intersection dimension.

We consider an S -brane solution governed by the function

$$\hat{H} = 1 + P\rho^2, \quad (2.5)$$

where ρ is a time variable,

$$P = \frac{1}{8} K Q^2, \quad (2.6)$$

and

$$K = K_s = d(I_s) \left(1 + \frac{d(I_s)}{2-D} \right) + \lambda_{s\alpha} \lambda_{s\beta} h^{\alpha\beta}, \quad (2.7)$$

$s = 1, 2$, is supposed to be non zero. Here $(h^{\alpha\beta}) = (h_{\alpha\beta})^{-1}$. Thus $K_1 = K_2 = K$.

The intersection rule is as follows:

$$d(I_1 \cap I_2) = \frac{d(I_1)d(I_2)}{D-2} - \lambda_{1\alpha} \lambda_{2\beta} h^{\alpha\beta} - \frac{1}{2} K. \quad (2.8)$$

This relation corresponds to the Lie algebra A_2 [21, 22]. Recall that $K_s = (U^s, U^s)$, $s = 1, 2$, where the

“electric” U^s vectors and the scalar products were defined in [23, 24] (see also [25, 21]). The relations $K_1 = K_2$ and (2.8) follow just from the formula $(A_{ss'}) = (2(U^s, U^{s'}) / (U^{s'}, U^{s'}))$, where $(A_{ss'})$ is the Cartan matrix for A_2 (with $A_{12} = A_{21} = -1$).

We consider the following exact solution:

$$g = \hat{H}^{2A} \left\{ -d\rho \otimes d\rho + \hat{H}^{-4B} (\rho^2 g^1 + g^2) + \hat{H}^{-2B} g^3 + \hat{H}^{-2B} g^4 + g^5 \right\}, \quad (2.9)$$

$$\exp(\varphi^\alpha) = \hat{H}^{B\lambda_1^\alpha + B\lambda_2^\alpha}, \quad (2.10)$$

$$F^1 = -Q \hat{H}^{-2} \rho d\rho \wedge \tau_1 \wedge \tau_2 \wedge \tau_3, \quad (2.11)$$

$$F^2 = -Q \hat{H}^{-2} \rho d\rho \wedge \tau_1 \wedge \tau_2 \wedge \tau_4, \quad (2.12)$$

where

$$A = 2K^{-1} \sum_{s=1,2} \frac{d(I_s)}{D-2}, \quad (2.13)$$

$$B = 2K^{-1}, \quad (2.14)$$

$s = 1, 2$. Here τ_i denotes a volume form on M_i ($g_1 = dx \otimes dx$, $\tau_1 = dx$). We remind the reader that all Ricci-flat metrics g^1, \dots, g^5 have Euclidean signatures.

This solution is a special case of a more general solution from [26] corresponding to the Lie algebra A_2 . It may also be obtained as a special 1-block case of S -brane solutions from [28].

3. Solutions with acceleration

Let us introduce the synchronous time variable $\tau = \tau(\rho)$ by the following relation:

$$\tau = \int_0^\rho d\bar{\rho} [\hat{H}(\bar{\rho})]^A \quad (3.1)$$

We put $P < 0$, and hence due to (2.6) $K < 0$ which implies $A < 0$. Let us consider two intervals of the parameter A :

$$(i) \quad A < -1, \quad (3.2)$$

$$(ii) \quad -1 < A < 0. \quad (3.3)$$

In case (i), the function $\tau = \tau(\rho)$ is monotonically increasing from 0 to $+\infty$, for $\rho \in (0, \rho_1)$, where $\rho_1 = |P|^{-1/2}$, while in case (ii) it is monotonically increasing from 0 to a finite value $\tau_1 = \tau(\rho_1)$.

Let the space M_5 be “our” 3-dimensional space with the scale factor

$$a_5 = \hat{H}^A. \quad (3.4)$$

For the first branch (i), we get the asymptotic relation

$$a_5 \sim \text{const } \tau^\nu, \quad (3.5)$$

for $\tau \rightarrow +\infty$, where

$$\nu = A/(A+1) \quad (3.6)$$

and, due to (3.2), $\nu > 1$. For the second branch (ii) we obtain

$$a_5 \sim \text{const } (\tau_1 - \tau)^\nu, \quad (3.7)$$

for $\tau \rightarrow \tau_1 - 0$, where $\nu < 0$ due to (3.3), see (3.6).

Thus we get an asymptotic accelerated expansion of the 3D factor space M_5 in both cases (i) and (ii), and $a_5 \rightarrow +\infty$.

Moreover, it may be readily verified that the accelerated expansion takes place for all $\tau > 0$, i.e.,

$$\dot{a}_5 > 0, \quad \ddot{a}_5 > 0. \quad (3.8)$$

Here and in what follows we denote $\dot{f} = df/d\tau$.

Indeed, using the relation $d\tau/d\rho = \hat{H}^A$ (see (3.1)), we get

$$\dot{a}_5 = \frac{d\rho}{d\tau} \frac{da_5}{d\rho} = \frac{2|A||P|\rho}{\hat{H}}, \quad (3.9)$$

and

$$\ddot{a}_5 = \frac{d\rho}{d\tau} \frac{d}{d\rho} \frac{da_5}{d\rho} = \frac{2|A||P|}{\hat{H}^{2+A}} (1 + |P|\rho^2), \quad (3.10)$$

that certainly implies the inequalities in (3.8).

Now we consider the variation of the effective G . For our model, the 4-dimensional gravitational “constant” (in Jordan’s frame) is

$$G = \text{const} \cdot \prod_{i=1}^4 (a_i^{-d_i}) = \hat{H}^{2A} \rho^{-1}, \quad (3.11)$$

where

$$\begin{aligned} a_1 &= \hat{H}^{A-2B} \rho, & a_2 &= \hat{H}^{A-2B}, \\ a_3 &= a_4 = \hat{H}^{A-B} \end{aligned} \quad (3.12)$$

are the scale factors of the “internal” spaces M_1, \dots, M_4 , respectively.

The function $G(\tau)$ has a minimum at the point τ_0 corresponding to

$$\rho_0 = \frac{|P|^{-1}}{1 + 4|A|}. \quad (3.13)$$

At this point, \dot{G} is zero. This follows from an explicit relation for the dimensionless variation of G ,

$$\delta = \dot{G}/(GH) = 2 + \frac{1 - |P|\rho^2}{2A|P|\rho^2}, \quad (3.14)$$

where

$$H = \frac{\dot{a}_5}{a_5} \quad (3.15)$$

is the Hubble parameter of our space.

The function $G(\tau)$ monotonically decreases from $+\infty$ to $G_0 = G(\tau_0)$ for $\tau \in (0, \tau_0)$ and monotonically increases from G_0 to $+\infty$ for $\tau \in (\tau_0, \bar{\tau}_1)$. Here $\bar{\tau}_1 = +\infty$ for the case (i) and $\bar{\tau}_1 = \tau_1$ for the case (ii).

The scale factors $a_2(\tau), a_3(\tau), a_4(\tau)$ monotonically decrease from 1 to 0 for $\tau \in (0, \bar{\tau}_1)$ since the powers

$A - B$ and $A - 2B$ are positive and $P < 0$. The scale factor $a_1(\tau)$ monotonically increases from zero to $a_1(\tau_2)$ for $\tau \in (0, \tau_2)$ and monotonically decreases from $a_1(\tau_2)$ to zero for $\tau \in (\tau_2, \bar{\tau}_1)$, where τ_2 is a point of maximum.

We should consider only solutions with accelerated expansion of our space and small enough variations of the gravitational constant obeying the present experimental constraint

$$|\delta| < 0.1. \quad (3.16)$$

Here, as in the model with two curvatures [12], τ is restricted to a certain range containing τ_0 . It follows from (3.14) that in the asymptotical regions (3.5) and (3.7) $\delta \rightarrow 2$, which is unacceptable due to the experimental bounds (3.16). This restriction is satisfied for a range containing the point τ_0 where $\delta = 0$.

A calculation of \dot{G} in the linear approximation near τ_0 gives the following approximate relation for the dimensionless parameter of relative variation of G :

$$\delta \approx (8 + 2|A|^{-1})H_0(\tau - \tau_0), \quad (3.17)$$

where $H_0 = H(\tau_0)$ (compare with an analogous relation in [12]). This relation gives approximate bounds for values of the time variable τ allowed by the restriction on \dot{G} . (For another mechanism of obtaining an acceleration in a multidimensional model with a “perfect fluid” see [13, 14]).

It should be stressed that the solution under consideration with $P < 0$, $d_1 = 1$ and $d_5 = 3$ takes place when the configuration of branes, the matrix $(h_{\alpha\beta})$ and the dilatonic coupling vectors λ_a , obey the relations (2.7) and (2.8) with $K < 0$. This is not possible when $(h_{\alpha\beta})$ is positive-definite, since in this case $K > 0$. In the next section we will give an example of a setup obeying (2.7) and (2.8) by introducing “phantom” fields.

4. Example

Let us consider the following example: $N_1 = N_2 = N$, i.e. the ranks of forms are equal, and $l = 2$, $(h_{\alpha\beta}) = -(\delta_{\alpha\beta})$, i.e. there are two “phantom” scalar fields. We also put $d_3 = d_4$. Due to (2.3), $d(I_1) = d(I_2) = N - 1 = 1 + d_2 + d_3$.

Then the relations (2.7) and (2.8) read

$$\vec{\lambda}_1^2 = \vec{\lambda}_2^2 = (N - 1)\left(1 + \frac{N - 1}{2 - D}\right) - K, \quad (4.1)$$

and

$$\vec{\lambda}_1 \vec{\lambda}_2 = 1 + d_2 - \frac{(N - 1)^2}{D - 2} + \frac{1}{2}K, \quad (4.2)$$

where $K < 0$. Here we have used that $d(I_1) = d(I_2) = N - 1$ and $d(I_1 \cap I_2) = 1 + d_2$.

The relations (4.1) and (4.2) are compatible since it may be verified that they imply

$$\frac{\vec{\lambda}_1 \vec{\lambda}_2}{|\vec{\lambda}_1||\vec{\lambda}_2|} \in (-1, +1) \quad (4.3)$$

i.e., the vectors $\vec{\lambda}_1$ and $\vec{\lambda}_2$, belonging to the Euclidean space \mathbb{R}^2 , and obeying the relations (4.1) and (4.2), do exist. The left-hand side of Eq. (4.3) gives $\cos \theta$, where θ is the angle between these two vectors.

The solution from Sec. 2 for the metric and two phantom fields in this special case reads

$$g = \hat{H}^{2A} \left\{ -d\rho \otimes d\rho + \hat{H}^{-8/K} (\rho^2 g^1 + g^2) + \hat{H}^{-4/K} (g^3 + g^4) + g^5 \right\}, \quad (4.4)$$

$$\exp(\varphi^\alpha) = \hat{H}^{-2(\lambda_{1\alpha} + \lambda_{2\alpha})/K}, \quad (4.5)$$

where

$$A = \frac{4(N-1)}{K(D-2)}. \quad (4.6)$$

The relations for the form fields (2.11) and (2.12) remain the same. Recall that $\hat{H} = 1 + \frac{1}{8} K Q^2 \rho^2$, $K < 0$. The relations (4.1) and (4.2) for the scalar products of dilatonic coupling vectors are assumed. Recall that all factor spaces (M_i, g^i) are Ricci-flat and $d_i = \dim M_i$ with $d_1 = 1$ and $d_5 = 3$. Hence the spaces (M_1, g^1) and (M_5, g^5) are flat.

Here, as in the previous section, the metric (4.4) describes an accelerated expansion of the fifth factor space (M_5, g^5) for $A \neq -1$. The 4D effective gravitational “constant” $G(\tau)$, as a function of the synchronous time variable, monotonically decreases from $+\infty$ to its minimum value $G_0 = G(\tau_0)$ for $\tau \in (0, \tau_0)$ and monotonically increases from G_0 to $+\infty$ for $\tau \in (\tau_0, \bar{\tau}_1)$. Recall that $\bar{\tau}_1$ is finite for $-1 < A < 0$ and $\bar{\tau}_1 = +\infty$ for $A < -1$. There exists a time interval (τ_-, τ_+) containing τ_0 where the variation of $G(\tau)$ obeys the experimental bound (3.16).

5. Conclusions

We have considered an S -brane solution with two non-composite electric branes and a set of l scalar fields. The solution contains five factor spaces, and the fifth one, M_5 , is interpreted as “our” 3D space.

As in the of the model with two non-zero curvatures [12], we have found that there exists a time interval where accelerated expansion of “our” 3-dimensional space co-exists with a small enough value of \dot{G}/G obeying the experimental bounds.

Other results on G , \dot{G} and $G(r)$ may be found in [29, 30, 31, 32, 33, 34].

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